ADP-05-01/T611

Neutral pion decay into $\nu\bar{\nu}$ in dense skyrmion matter

Alexander C. Kalloniatis[a],* Jonathan D. Carroll[a],[†] and Byung-Yoon Park[a,b‡]

[a] CSSM, University of Adelaide, Adelaide 5005, Australia
[b] Department of Physics,Chungnam National University, Daejon 305-764, Korea

(Dated: February 2, 2008)

We study the weak decay of the neutral pion to a neutrino-antineutrino pair, $\pi^0 \to \nu\bar{\nu}$, in the Skyrme model. In baryon free-space the process is forbidden by helicity while in a dense baryonic medium, the process becomes possible already to leading order in G_F due to the break-down of Lorentz symmetry in the background medium.

PACS numbers: 12.39.Dc,13.20.Cz,26.60.+c

I. INTRODUCTION

Neutrinos and their means of production play an important role in cosmological and astrophysical phenomena [1]. In particular in this paper we shall be concerned with the weak emission of a neutrino-antineutrino pair from a neutral pion, $\pi^0 \to \nu\bar{\nu}$, which figures centrally in the pion pole mechanism [2, 3] for the process $\gamma\gamma \to \pi^0 \to \nu\bar{\nu}$ relevant to the neutrino emission process $\gamma\gamma \to \nu\bar{\nu}$. The latter has been suggested as a competitor to the modified URCA process for neutron star cooling for most densities and for temperatures around 10^9 K [4, 5] while it has been shown to be forbidden to lowest order in G_F by Gell-Mann [6]. On the other hand, medium modifications of the pion can also influence the modified URCA processes, as shown in [7], where neutrino-pair radiation from nucleons in π^0-condensed nucleon matter was studied. However, [8] has argued that such modifications may not be so efficient because of the gap between the energy bands of nucleons interacting with the periodic structure of the condensed pion field.

Returning then to the pion-pole mechanism, of course in the limit of zero neutrino mass the amplitude for the corresponding process vanishes for local weak currents of the $V - A$ form. In [3] it was shown that a deviation from vanishing neutrino mass could lead to small neutrino emission rates suppressed by a factor $(m_\nu/m_\pi)^3$, which however can be relevant in astrophysical processes for temperatures where the pion pole is significant. In [9], the pion pole mechanism is used to determine a bound on the branching ratio of $R < 2.9 \times 10^{-13}$ for the decay $\pi^0 \to \nu\bar{\nu}$, as well as an upper bound on the neutrino mass as $m_\nu < 420$ keV. In [10], various medium effects in the pion-pole mechanism $\gamma\gamma \to \pi^0 \to \nu\bar{\nu}$ have been discussed. In particular, the nuclear medium effects on the pion propagator are estimated to determine the nuclear matter to vacuum *ratio* of energy loss from pion-pole neutrino emission, such that the decay width $\Gamma(\pi^0 \to \nu\bar{\nu})$ is cancelled but assumed to be non-zero either from a neutrino mass or some other mechanism. The enhancement seen in this ratio of energy loss rates for densities twice that of nuclear matter is of the order of 10^4 coming from the pion propagator modifications in dense nuclear matter alone. In this paper we shall consider precisely the baryonic medium modifications of $\pi^0 \to \nu\bar{\nu}$, adding to the picture established thus far by [10].

The simple reason that $\pi^0 \to \nu\bar{\nu}$ is forbidden in the limit of massless neutrino is the helicity selection rule. Two outgoing neutrinos have opposite chirality, which combine to total angular momentum one in the rest frame of the decaying meson. Thus, it is impossible for the pseudoscalar pion to decay into $\nu\bar{\nu}$. However in a dense medium such a process has been shown to be possible [7, 11, 12]. Of course, a scalar meson at rest with respect to the dense medium cannot decay into $\nu\bar{\nu}$ pairs. However, a background medium provides an explicit reference frame so a scalar meson moving with respect to the medium can have higher angular momentum admixtures in its wavefunction. In [11] neutrino emissivity is studied in the colour-flavour locked (CFL) phase at extreme densities via decay of the "generalised pion" which is the lightest Goldstone boson in this phase. The nonvanishing decay comes from a breakdown of Lorentz symmetry which is explicitly incorporated into an effective Lagrangian via differing temporal and spatial components of the pion decay constant, f_T and $f_S = f_T v_\pi^2$ where v_π^2 represents an "in-medium pion velocity". The resulting neutrino pair emissivity from π^0 decay does not vanish and is proportional to $(f_T - f_S)^2$.

In this work, we study then the decay of the neutral pion to $\nu\bar{\nu}$ specifically in a dense baryonic medium, namely we work at densities below the critical point for restoration of chiral symmetry. As in previous works [13, 14, 15, 16, 17], we describe the baryonic matter via the Skyrme model where both pions and dense baryonic matter are described by a single effective chiral Lagrangian. The basic strategy of the approach begins with the Skyrme conjecture [18] that a soliton (skyrmion) of the meson Lagrangian can be taken as a baryon so that dense baryonic matter can be approximated as a system of infinitely many skyrmions. Pions can be incorporated either as fluctuations on the baryon free vacuum or on a

*Electronic address: akalloni@physics.adelaide.edu.au
†Electronic address: jcarroll@physics.adelaide.edu.au
‡Electronic address: bypark@cnu.ac.kr

classical field configuration describing dense matter. The dynamics of the pions become then strongly dependent on the background properties such as the baryon density. If we accept with [19] that the Skyrme model can be applied up to some density, its unique feature of a unified meson-baryon description provides an elegant framework for investigating nonperturbative meson properties in dense baryonic matter. In particular, we need not assume any density dependence of the in-medium parameters but rather work with a single model Lagrangian whose parameters are fixed for mesons in free space. Only the classical ground state solution describing the dense skyrmion matter becomes density dependent as do naturally in turn the fluctuating mesons on top of this dense baryonic background.

The weak vector bosons are incorporated into the model Lagrangian by standard gauging. In baryon free space the $\pi^0 - Z$ vertex from the kinetic term of the Lagrangian leads to a trivially vanishing amplitude for the process $\pi^0 \to Z \to \nu\bar{\nu}$. A new vertex for $\pi^0 \to ZZ$ arises from the gauging of the WZW term. This also yields a vanishing result for $\pi^0 \to ZZ \to \nu\bar{\nu}$ however at the level of the amplitude-squared, which we shall show in this paper. Most significantly we show in this paper that for neutral pion fluctuations in dense skyrmion matter the dynamics of the pions are affected by the background and the $\pi^0 \to \nu\bar{\nu}$ can occur already to order G_F with a strength determined by nonperturbative dynamics to be of order 10^{-3}, significantly larger than the $(m_\nu/m_\pi)^3$ behaviour for keV neutrinos. As in [11, 12], the non-vanishing amplitude is ascribed to pion dynamics in the presence of the breakdown of Lorentz symmetry however now in the *hadronic* rather than the CFL phase. Unlike [7] which also works in the hadronic phase, in the present work the nuclear medium will modify the pion, closer in spirit to [10] where nevertheless these influences on $\pi^0 \to \nu\bar{\nu}$ were not considered in the hadronic phase.

A difference between our approach and that of [11, 12] is that our initial Lagrangian itself respects Lorentz invariance; the breaking of the symmetry rather takes place at the level of the background field *ansatz* over which the pions fluctuate. In view of the assumptions behind the skyrmionic model for baryonic matter, our results are qualitative but are summarised in Fig.4 where enhancement as a function of density of the decay $\pi^0 \to \nu\bar{\nu}$ is indicated in the quantity δ, which is also intimately related to the in-medium velocity of the pion v_π. We compare these specific results to those of [11] as well as to our previous work on v_π in [15].

This paper is organised as follows: in the next section we introduce the model Lagrangian, followed by the demonstration that $\pi^0 \to \nu\bar{\nu}$ vanishes in free space. In section IV we analyse the process in dense skyrmion matter and give a brief statement of conclusions at the end.

II. MODEL LAGRANGIAN

We work with the Skyrme model Lagrangian [18]

$$\begin{aligned}\mathcal{L} &= \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \mathcal{L}_{\rm sk} \\ &\quad + \frac{f_\pi^2 m_\pi^2}{4}\mathrm{Tr}(U^\dagger + U - 2) + \mathcal{L}_{\rm WZW} \qquad (1)\end{aligned}$$

where $U = \exp(i\vec{\tau}\cdot\vec{\pi}/f_\pi) \in SU(2)$, m_π and f_π are respectively the pion mass and decay constant in free space. The Skyrme term $\mathcal{L}_{\rm sk}$ is a higher derivative term introduced into the Lagrangian to stabilise the soliton solution, namely

$$\mathcal{L}_{\rm sk} = \frac{1}{32e^2}\mathrm{Tr}\left([L_\mu, L_\nu]^2\right), \qquad (2)$$

where $L_\mu \equiv (\partial_\mu U)U^\dagger$ and where $R_\mu \equiv U^\dagger(\partial_\mu U)$ will be later required. Finally, the Wess-Zumino-Witten (WZW) term $\mathcal{L}_{\rm WZW}$ is necessary to break the symmetry of Eq.(1) under $U \to U^\dagger$ which is not a genuine symmetry of QCD. The corresponding action can be written locally as [20]

$$\Gamma_{\rm WZW}(U) = C\int_{M^5}\mathrm{Tr}(\hat{L}^5) = C\int_{M^5}\mathrm{Tr}(\hat{R}^5) \qquad (3)$$

in a five-dimensional space M^5 whose boundary is ordinary space and time. The constant C is determined as $C = -\frac{iN_c}{240\pi^2}$ so that the anomalous Lagrangian is consistent with the triangle anomaly for the process $\pi^0 \to \gamma\gamma$ when the number of colors $N_c = 3$. For compactness in the following we adopt the differential one-form notation with

$$\hat{L} = L_\mu dx^\mu = (dU)U^\dagger,$$

and

$$\hat{R} = U^\dagger dU = U^\dagger \hat{L} U. \qquad (4)$$

Gauging all but the WZW part of the Lagrangian enables the electroweak vector fields to be introduced via minimal coupling:

$$D_\mu U = \partial_\mu U - i\mathcal{A}_\mu^L U + U(i\mathcal{A}_\mu^R), \qquad (5)$$

where

$$\begin{aligned}\mathcal{A}_\mu^L &= eQA_\mu - \frac{g}{\sqrt{2}}(W_\mu^+\tau^+ + W_\mu^-\tau^-) \\ &\quad + \frac{g}{2\cos\theta_W}Z_\mu(\tau^3 - 2Q\sin^2\theta_W), \qquad (6) \\ \mathcal{A}_\mu^R &= eQA_\mu - \frac{g}{2\cos\theta_W}Z_\mu(2Q\sin^2\theta_W). \qquad (7)\end{aligned}$$

Here, A_μ is the electromagnetic field which couples with strength $Q = \mathrm{diag}(2/3, -1/3)$, $W_\mu^{+,-}$ and Z_μ are the $SU(2)$ electroweak gauge fields, g is the weak coupling constant, θ_W is the Weinberg angle, and $\tau^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2)$ with τ^a the Pauli matrices.

In contrast, as is well-known, minimal substitution does not work in gauging the WZW action. The so-called trial and error Noether method [20] gives the gauged WZW action

$$
\begin{aligned}
&\tilde{\Gamma}_{\rm WZW}(U,\mathcal{A}^L_\mu,\mathcal{A}^R_\mu)\\
&\quad=\Gamma_{\rm WZW}(U)+5Ci\int_{M^4}\mathrm{Tr}(\hat{\mathcal{A}}^L\hat{L}^3+\hat{\mathcal{A}}^R\hat{R}^3)\\
&\qquad-5C\int_{M^4}\mathrm{Tr}\big((d\hat{\mathcal{A}}^L\hat{\mathcal{A}}^L+\hat{\mathcal{A}}^Ld\hat{\mathcal{A}}^L)\hat{L}\\
&\qquad\qquad+(d\hat{\mathcal{A}}^R\hat{\mathcal{A}}^R+\hat{\mathcal{A}}^Rd\hat{\mathcal{A}}^R)\hat{R}\big)\\
&\qquad+5C\int_{M^4}\mathrm{Tr}\big(d\hat{\mathcal{A}}^L dU\hat{\mathcal{A}}^RU^\dagger-d\hat{\mathcal{A}}^Rd(U^\dagger)\hat{\mathcal{A}}^LU\big)\\
&\qquad+\cdots.
\end{aligned}
\tag{8}
$$

Here we have shown only the terms relevant for the processes involving single neutral pion and one or two gauge bosons. A complete list of terms can be found in [20].

The leptonic part of the Lagrangian is

$$
\begin{aligned}
\mathcal{L}^{\rm lept.}_{Z\nu\bar\nu} &= -\frac{g}{2\cos\theta_W}Z_\mu\bar\nu_e\gamma_\mu\frac{1-\gamma_5}{2}\nu_e+\bar\nu_e(i\partial\!\!\!/)\nu_e\\
&\quad+\big(e\to\mu,\tau\ \text{terms}\big).
\end{aligned}
\tag{9}
$$

Here again, we have presented only the relevant terms for the process of our concern, that is, $Z\to\nu\bar\nu$. The neutrino field ν_e is constrained to the left-handed one; $\gamma_5\nu_e=-\nu_e$.

Finally, the Lagrangian should be complemented by the terms for the dynamics of the gauge bosons, namely

$$
\begin{aligned}
\mathcal{L}_{\gamma,Z,W^\pm} &= -\frac14\mathrm{Tr}(F^{\mathcal{A}^L}_{\mu\nu}F^{\mathcal{A}^L\mu\nu}+F^{\mathcal{A}^R}_{\mu\nu}F^{\mathcal{A}^R\mu\nu})\\
&\quad-\frac12\big(M_W^2(W^+_\mu W^{+\mu}+W^-_\mu W^{-\mu})\\
&\qquad+M_Z^2Z_\mu Z^\mu\big),
\end{aligned}
\tag{10}
$$

where $F^{\mathcal{L},\mathcal{R}}_{\mu\nu}$ are field strength tensors and M_W and M_Z are the masses of the gauge bosons.

III. $\pi^0\to\nu\bar\nu$ IN BARYON-FREE SPACE

The lowest static energy configuration for baryon free space is simply a constant U. Without loss of a generality we can set $U=1$. Thus, the Lagrangian constructed in the previous section governs the dynamics of the pions in baryon free space as it is. By expanding $U=\exp(i\vec\tau\cdot\vec\pi/f_\pi)$ up to a given order in the pion field we obtain the hadronic part of the lagrangian for the weak decay of the neutral pion as

$$
\begin{aligned}
\mathcal{L} &= \tfrac12\partial_\mu\pi^a\partial^\mu\pi^a-\tfrac12 m_\pi^2\pi^a\pi^a\\
&\quad+\frac{f_\pi g}{2\cos\theta_W}Z_\mu\partial^\mu\pi^0\\
&\quad-\frac{N_c}{48\pi^2f_\pi}\pi^0\tan^2\theta_W\cos2\theta_W g^2\varepsilon^{\mu\nu\alpha\beta}\partial_\mu Z_\nu\partial_\alpha Z_\beta\\
&\quad+\cdots.
\end{aligned}
\tag{11}
$$

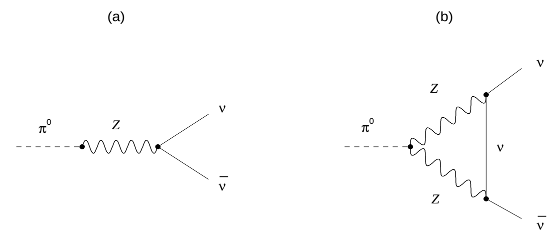

FIG. 1: $\pi^0\to\nu\bar\nu$ processes.

The Lagrangian yields proper vertices for $\pi^0\to Z$ and $\pi^0\to ZZ$, which contribute to the processes $\pi^0\to\nu\bar\nu$ as shown in Fig.1. Observe that there does not appear a term such as $\pi^0\to W^+W^-$ despite fulfilling charge conservation.

The amplitude for the process shown in Fig.1a is

$$
\mathcal{M}_{\pi^0\to Z\to\nu\bar\nu}=\frac{G_Ff_\pi}{\sqrt2}\bar u_\nu(p_1)p\!\!\!/(1-\gamma_5)v_\nu(p_2),\tag{12}
$$

where we have approximated the Z-boson propagator simply as $i/(p^2-M_Z^2)\sim -i/M_Z^2$ and $G_F=g^2/(8M_W^2)$. $p_1=(\omega_1,\vec p_1)$ and $p_2=(\omega_2,-\vec p_2)$ are the energy-momenta of outgoing neutrinos and $p=p_1+p_2$ is that of the incoming pion. For massless neutrinos $\bar u_\nu(p_1)p\!\!\!/_1=p\!\!\!/_2v(p_2)=0$ so that the amplitude vanishes identically.

The process of Fig.1b leads to

$$
\begin{aligned}
\mathcal{M}_{\pi^0\to ZZ\to\nu\bar\nu} &= -i\frac{N_cg^2}{48\pi^2f_\pi}\tan^2\theta_W\cos2\theta_W\\
&\quad\times\varepsilon_{\mu\nu\alpha\beta}\int\frac{d^4k}{(2\pi)^4}k_1^\alpha k_2^\beta\\
&\quad\times\frac{\bar u(p_1)\gamma^\mu(1-\gamma_5)(k\!\!\!/_2-p\!\!\!/_2)\gamma^\nu(1-\gamma_5)v(p_2)}{(k_1^2-M_Z^2)(k_2^2-M_Z^2)(k_2-p_2)^2}
\end{aligned}
\tag{13}
$$

where $k_1=k+p/2$, $k_2=-k+p/2$ represent the momenta of the two internal Z-boson lines. The loop-momenta are integrated only up to a cut-off $\Lambda\sim 1$GeV for the low-energy effective theory. Taking the square of the amplitude and tracing over the spinor structure gives

$$
\begin{aligned}
|\mathcal{M}_{\pi^0\to ZZ\to\nu\bar\nu}|^2 &= \frac{N_c^2g^4}{144\pi^4f_\pi^2}\tan^4\theta_W\cos^2 2\theta_W\\
&\quad\times I_\Lambda(p,p_1,p_2)(p^2p_1\cdot p_2-2p\cdot p_1p\cdot p_2)
\end{aligned}
\tag{14}
$$

where $I_\Lambda(p,p_1,p_2)$ comes from the loop-integration but whose form is not important in view of the next result: in the pion rest frame $(p^2p_1\cdot p_2-2p\cdot p_1p\cdot p_2)$, which arises from the spinor traces for massless neutrinos, vanishes identically, consistent with the helicity selection rule.

IV. $\pi^0\to\nu\bar\nu$ IN THE DENSE SKYRMION MATTER

The Skyrme Lagrangian supports classical soliton solutions, skyrmions, whose topological winding number can

be interpreted as a baryon number. With this conjecture, dense baryonic matter can be approximated as a system of skyrmions. Let $U_0(\vec{r}) = n_0 + i\vec{\tau}\cdot\vec{n}$ parametrise the lowest energy configuration for a given baryon number density, where n_0, $\vec{n}$ are space-dependent functions describing the ensemble of skyrmions. Then we can incorporate quantum fluctuations on top of this classical ground state configuration via the *ansatz*

$$U = \sqrt{U_\pi} U_0 \sqrt{U_\pi}, \tag{15}$$

where $U_\pi = \exp(i\vec{\tau}\cdot\vec{\pi}/f_\pi)$. The trivial case of $U_0(\vec{r}) = 1$ corresponds to pions in baryon-free space. For the skyrmion matter encoded in $U_0(\vec{r})$ we take the crystal configurations studied in [19]. At low density, the skyrmion matter is arranged in a face-centered-cubic crystal where a well-localised single skyrmion occupies each lattice site. As the density increases, the skyrmions begin to overlap. At higher density, the phase becomes the so-called "half-skyrmion cubic crystal", where about half of the baryon number of the original skyrmion leaks out to generate another well-defined dense object in the region of overlap. When the skyrmion matter is in an exact half-skyrmion phase, the spatial average value of the $U_0(\vec{r})$ vanishes identically.

Substituting the *ansatz* into the Lagrangian and expanding in fluctuations, we obtain a number of structures. In the following we explicitly give only those terms governing the dynamics of the fluctuations in the presence of the background potentials which will be relevant to our discussion. Thus we obtain

$$\begin{aligned}\mathcal{L} \;=\; & \tfrac{1}{2}G_{ab}(\vec{r})\partial_\mu\pi^a\partial^\mu\pi^b - \tfrac{1}{2}n_0 m_\pi^2\pi^a\pi^a \\ & +\tfrac{1}{2}\varepsilon^{abc}\pi^a\partial_i\pi^b V_i^c(\vec{r}) \\ & +\frac{f_\pi g}{2\cos\theta_W}G_{33}(\vec{r})Z^\mu\partial_\mu\pi^0 \\ & +\frac{N_c}{24\pi^2}\frac{1}{f_\pi}\frac{g\sin^2\theta}{\cos\theta_W}\varepsilon^{\mu\nu\alpha\beta}Z_\mu\partial_\nu\pi^0 H_{\alpha\beta}(\vec{r}) \\ & +\cdots,\end{aligned} \tag{16}$$

where

$$G_{ab}(\vec{r}) = n_0^2\delta_{ab} + n_a n_b, \tag{17}$$

$$V_i^a(\vec{r}) = \varepsilon^{abc}n_b\partial_i n_c \tag{18}$$

$$\begin{aligned}H_{\alpha\beta}(\vec{r}) &= (n_0^2+n_3^2)(\partial_\alpha n_1\partial_\beta n_2 - \partial_\alpha n_2\partial_\beta n_1) \\ &\quad -(n_2\partial_\alpha n_1 - n_1\partial_\alpha n_2)\partial_\beta(n_0^2+n_3^2) \\ &= G_{33}\partial_\alpha V_\beta^3 + V_\beta^3\partial_\alpha G_{33}.\end{aligned} \tag{19}$$

We stress that the terms not given here govern the skyrmion dynamics summarised in these potentials, and have been treated in earlier works. In particular, formulae for $n_0, \vec{n}$ are given in Eqs.(9-12) of [13] where a different notation is used: $n_0 \leftarrow \bar{\sigma}, n_i \leftarrow \bar{\pi}_i$.

We now consider whether it is possible for the process $\pi^0 \to \nu\bar{\nu}$ to occur through a one boson intermediate state, which is leading order in G_F. To this end we need $G_{33}(\vec{r})$, which comes from the kinetic term of the Lagrangian, and $H_{\alpha\beta}$ which originates in the WZW terms that are linear in $\mathcal{A}_\mu^L$ or $\mathcal{A}_\mu^R$. The contribution of G_{33} is involved with the symmetric (in $\mu\nu$) structure $g^{\mu\nu}Z_\mu\partial_\nu\pi^0$ while $H_{\alpha\beta}$ appears with the antisymmetric (in $\mu\nu$) $\varepsilon^{\mu\nu\alpha\beta}Z_\mu\partial_\nu\pi^0$.

In a naive mean field approximation, we can replace the space-dependent potentials such as $G_{ab}(\vec{r})$ and $H_{\alpha\beta}(\vec{r})$ by their spatial averages $\langle G_{ab}(\vec{r})\rangle$ and $\langle H_{\alpha\beta}(\vec{r})\rangle$; *viz.*

$$\begin{aligned}\langle G_{ab}\rangle &\equiv \frac{1}{V}\int_V d^3x G_{ab}(\vec{r}) = G\delta_{ab}, \\ \langle V_i^a\rangle &= \langle H_{ab}\rangle = 0,\end{aligned} \tag{20}$$

where V is the volume of a unit cell of the skyrmion crystal over which the integration is carried out and G is a constant. Due to the symmetric structure of the crystal solution $U_0(\vec{r})$, these averages of the potentials $H_{\alpha\beta}(\vec{r})$ vanish. After renormalising the pion fields, $\pi^{*a} = \sqrt{G}\pi^a$, we obtain an *effective* Lagrangian as

$$\begin{aligned}\mathcal{L}^* \;=\; & \tfrac{1}{2}\partial_\mu\pi^{*a}\partial_\mu\pi^{*a} - \tfrac{1}{2}m_\pi^*\pi^{*a}\pi^{*a} \\ & +\frac{f_\pi^* g}{2\cos\theta_W}Z_\mu\partial^\mu\pi^{*0} \\ & +\cdots.\end{aligned} \tag{21}$$

Note that the structure of the Lagrangian is exactly the same as Eq.(11), that of the pions in baryon free space. Only the $\pi^0 \to Z$ vertex strength is modified by a factor $f_\pi^*/f_\pi = \sqrt{G}$. Lorentz symmetry is still conserved in this Lagrangian. Thus, the $\pi^0 \to Z$ vertex cannot contribute to $\pi^0 \to \nu\bar{\nu}$ as discussed in Sec. III.

In [15] is shown that Lorentz symmetry breakdown of pion dynamics in medium can be manifested only when higher order effects in the background potential beyond mean field approximation are incorporated. We now outline a perturbative expansion in the background potentials. We decompose the Lagrangian into an unperturbed part, $\mathcal{L}_{(0)}$ and an interaction part, $\mathcal{L}_I$. We take $\mathcal{L}_0$ be the Lagrangian for free pion in baryon free space, namely

$$\mathcal{L}_0 = \tfrac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a + \tfrac{1}{2}m_\pi^2\pi^a\pi^a, \tag{22}$$

and $\mathcal{L}_I$ the remaining terms in Eq.(16). The local potential $G_{ab}(\vec{r})$ in the kinetic term makes the quantization process somewhat nontrivial. Nevertheless the following steps are standard for perturbing in the spatial potentials - see [21]. The conjugate momenta of the pion fields π^a are given by

$$\Pi^a = \frac{\partial\mathcal{L}_0}{\partial\dot{\pi}^a} = \dot{\pi}^a \tag{23}$$

while the Hamiltonian is

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \tag{24}$$

where

$$\mathcal{H}_0 = \tfrac{1}{2}(\Pi^a\Pi^a + \partial_i\pi^a\partial_i\pi^a) + \tfrac{1}{2}m_\pi^2\pi^a\pi^a, \tag{25}$$

and $\mathcal{H}_I = -\mathcal{L}_I$.

This defines the pion propagator and the interaction Hamiltonian as schematically represented in Fig.2. The perturbation rule is now very simple:

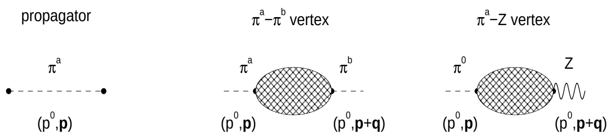

FIG. 2: Diagrammatic rules for the pion propagator and the interaction Hamiltonian.

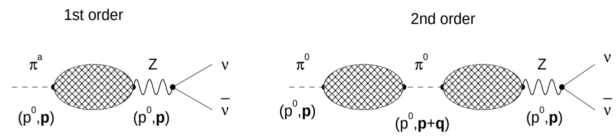

FIG. 3: First and second order diagrams in perturbation in the spatial potentials

1. Draw the diagrams for the corresponding physical process. The number of shaded blobs included in the diagram determines the order in perturbation.

2. For each internal pion propagator, we have
$$\int d^4p \, \frac{i}{(p^2 - m_\pi^2)}. \tag{26}$$

3. The $\pi^a(p) - \pi^b(p')$ vertex is given by
$$\begin{aligned}\delta^4(p + \vec{q} - p')\big\{ - (p^2 - \vec{p}\cdot\vec{q})(G_{ab}(\vec{q}) - \delta_{ab}) \\ + i\varepsilon^{abc}\vec{p}\cdot\vec{V}^c(\vec{q})\big\},\end{aligned} \tag{27}$$
where p and p' are the four-momenta of incoming and outgoing pions. We observe that the background medium is a source/sink for the additional momentum $\vec{q}$.

4. The $\pi^0 - Z$ vertex is
$$\delta^4(p + \vec{q} - p')\left\{ip_\mu G_{33}(\vec{q}) + i\varepsilon^{\mu\nu\alpha\beta}p_\mu H_{\alpha\beta}(\vec{q})\right\} \tag{28}$$

5. As for the external fields, we follow standard conventions.

The momentum components of the interactions are nothing but the Fourier expansion coefficients of the corresponding potentials, for example, defined as
$$G_{ab}(\vec{q}) = \frac{1}{V}\int_V d^3x e^{i\vec{q}\cdot\vec{x}} G_{ab}(\vec{x}), \tag{29}$$
with similar definitions for $V_i^a(\vec{q})$ and $H_{\alpha\beta}(\vec{q})$ applying. Due to the crystal structure of the background field configuration, only discrete values of $\vec{q}$ are allowed,
$$\vec{q} = \frac{2\pi}{L}\vec{m} \tag{30}$$
where $\vec{m}$ label the spatial positions of the Skyrmions in the FCC crystal and L is the lattice spacing [19]. The naive mean field approximation corresponds then to the $\vec{q} = 0$ components of objects such as $G_{ab}(\vec{q})$.

As for the $\pi^0 \to \nu\bar{\nu}$ process, we can have the diagrams shown in Fig.3 up to second order in the background potential. Clearly the baryonic medium can impart energy-momentum to the pion, which at the end of the day is why the process $\pi^0 \to \nu\bar{\nu}$ can proceed. However the general problem with arbitrary momentum imparted to the pion from the medium is difficult to solve. We shall restrict ourselves to those contributions which can be expressed in terms of a *local* $\pi^0\nu\bar{\nu}$ vertex, namely where the four-momentum of the incoming pion is the equal to the total momentum of the outgoing $\nu\bar{\nu}$. Certainly this is only a partial use of the information in the formalism. We will see that to leading order the effect will still be vanishing but at second order in the potentials a non-zero result can be obtained because of a reorganisation of the momentum in the system between the two interactions of the potentials.

In more detail then, the first order diagram reads
$$\mathcal{M}^{(1)} = \frac{G_F f_\pi}{\sqrt{2}} G_{33}(\vec{0})\bar{u}_\nu(p_1)\not{p}(1-\gamma_5)v_\nu(p_2), \tag{31}$$
which is the same as that for baryon free-space except for the factor $G_{33}(\vec{0}) = \langle G_{33}\rangle = G$. At any rate, this amplitude vanishes identically as hinted above because $\bar{u}_\nu(p_1)\not{p}(1-\gamma_5)v_\nu(p_2) = 0$ for momenta $p = p_1 + p_2$.

The second order diagram yields
$$\begin{aligned}\mathcal{M}^{(2)} &= \frac{G_F f_\pi}{\sqrt{2}} \sum_{\vec{q}} \frac{-(p^2 - \vec{q}\cdot\vec{p})G_{33}(\vec{q})}{(p+\vec{q})^2 - m_\pi^2} \\ &\quad \times \bar{u}_\nu(p_1)\big\{G_{33}(-\vec{q})(\not{p} + \not{q})(1-\gamma_5) \\ &\qquad + \varepsilon^{\mu\nu\alpha\beta}(p+\vec{q})_\mu H_{\alpha\beta}(-\vec{q})\gamma_\nu(1-\gamma_5)\big\}v_\nu(p_2) \\ &\approx \frac{G_F f_\pi}{\sqrt{2}}\delta\bar{u}_\nu(p_1)\not{\vec{p}}(1-\gamma_5)v_\nu(p_2),\end{aligned} \tag{32}$$
where
$$\delta = \tfrac{1}{3}\sum_{\vec{q}\neq 0}|G_{33}(\vec{q})|^2 \tag{33}$$
In the last step we have used that (i) $\bar{u}_\nu(p_1)\not{p}(1 - \gamma_5)v_\nu(p_2) = 0$, (ii) $|G_{33}(\vec{q})|^2$ is even with respect to $\vec{q} \leftrightarrow -\vec{q}$, and (iii) only the first nonvanishing term in the expansion the pion momentum point $(p_0, \vec{p}) = (m_\pi, \vec{0})$ need be kept. Note that only the spatial component $\vec{p}$ appears in the equation, which indicates that the Lorentz symmetry is indeed broken. Also, the contributions from $H_{\alpha\beta}$ of Eq.(16) only appear at higher order in this pion momentum expansion.

Thus, up to the second order in the background potential, we have the nonvanishing result
$$\sum_{\rm spin}|\mathcal{M}^{(1)+(2)}|^2 = 4G_F^2|\chi_{\rm Sk}|^2\omega_\pi^2(\omega_1\omega_2 + \vec{p}_1\cdot\vec{p}_2), \tag{34}$$

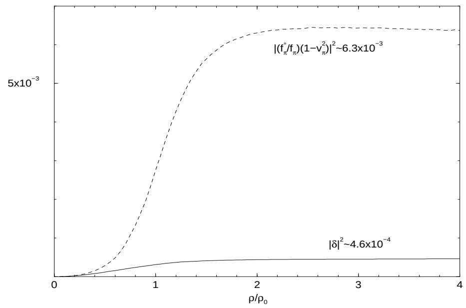

FIG. 4: Density dependence of δ (solid line) and the corresponding quantity involving the pion velocity from [15]. Here ρ_0 is the density of normal nuclear matter.

with $\chi_{\rm Sk} = f_\pi \delta + \mathcal{O}(m_\pi^2)$, $\omega_\pi = \sqrt{m_\pi^2 + v_\pi^2 p^2}$ the pion energy and $\omega_{1,2}$ the corresponding energy for the neutrino and anti-neutrino, the sum of which equals that for the pion. This is the result for one neutrino flavour; for more flavours the result is multiplied by N_ν.

Eqs.(33,34) are the main formal results of this work. We plot numerical results for the density dependence of δ in Fig.4. We now discuss this with respect to other work in the literature.

To begin with Eq.(34) is formally the same as the corresponding quantity in [11] but where for us $\chi_{\rm Sk}$ depends on the medium via δ and on the *vacuum* value for the decay constant f_π. However in [11] $\chi_{\rm Sk}$ is replaced by a quantity which we shall designate χ_{JPS} itself depending on the velocity in medium of the CFL generalised pion, v_π. This velocity in [11] measures in turn the breakdown of Lorentz symmetry as reflected in the non-equivalence of spatial (f_S) and temporal (f_T) components of the corresponding pion decay constant in medium,

$$|\chi_{JPS}| = |f_T - f_S| = |f_T(1 - v_\pi^2)|. \tag{35}$$

Thus despite studying quite different phases (hadronic vs CFL) we find formal agreement to second order in the potentials, up to what appears as a discrepancy: the absence of a medium generated distinction between components of the decay constant. However in a study of the pion propagator in dense skyrmionic matter in [15] it was shown by resummation of the corresponding expansion in potentials to infinite order that the in-medium pion velocity is given by

$$1 - v_\pi^2 = \sum_{\vec{q}\neq 0} \frac{\displaystyle\sum_{a=1,2} |V_a^3(\vec{q})|^2 + \frac{1}{3}\vec{q}^2 \sum_{a=1,2,3} |G_{3a}(\vec{q})|^2}{q^2}. \tag{36}$$

Comparing this to the present result for δ, Eq.(33), one expects that a similar, though technically more difficult, resummation in the context of $\pi^0 \to \nu\bar{\nu}$ will reveal the dependence of this decay in medium on the pion velocity and thus in turn on the inequivalence of f_T and f_S in medium. It is important to stress that the isospin mixing terms in the numerator of Eq.(36) cannot appear in our present calculation of Eq.(33) to second order in potentials, and can only occur after resummation. The analogous combination of in-medium quantities, f_π^* and v_π as obtained in [15] are indicated with the dashed curve in Fig.4 for comparison.

We see from Fig.4 our present result $|\delta|^2 \sim 4.6 \times 10^{-4}$ representing a significant density enhancement in the hadronic phase in the decay rate of $\pi^0 \to \nu\bar{\nu}$ as compared to $(m_\nu/m_\pi)^3 \sim 10^{-9}$ for keV neutrinos. Resummation effects, were they to mirror the results of [15], would amplify this further. In [15], the pion velocity is obtained in the range of $0.8 < v_\pi \leq 1$. This leads to a larger $1 - v_\pi^2$ than the $|\delta|$ obtained in our second order calculation, but is still smaller than that for the CFL phase in [11] where $v_\pi^2 = 1/3$.

V. CONCLUSION

We have studied the neutral pion decay into a neutrino-antineutrino pair. In baryon free space, the process is forbidden by helicity conservation for massless neutrinos. Though new vertices $\pi^0 \to Z$ and $\pi^0 \to ZZ$ emerge when the Skyrme Lagrangian is gauged, they also lead to a null result but for the square of the amplitude. On the other hand, we have shown that Lorentz symmetry breaks down due to the absolute frame of a background dense medium enabling $\pi^0 \to \nu\bar{\nu}$ which plays a crucial role in astrophysical phenomena.

Our result formally matches with [11], which however dealt with the colour-flavour-locked phase of matter at very high densities. Thus our work shows that such a pion pole mechanism already can be manifest even in the lower density hadronic phase.

Our result is the first study of the $\pi^0 \to \nu\bar{\nu}$ in Skyrme models, which elegantly combine meson and baryon dynamics. The limitations of the Skyrme model, in particular the provisional use of the unrealistic Skyrme crystal, mean our result is still qualitative. We stress though that the discrete translational symmetry of the crystal plays no role in these results, only its general isotropy which is anyway realistic. The mechanism for the decay is precisely the same as that hypothesised for the CFL phase and already can be exhibited for massless neutrinos in the hadronic phase. To the order in perturbation in background potentials studied here and retaining only those contributions to an effective local $\pi^0\nu\bar{\nu}$ vertex, the result formally agrees with that in the CFL phase. Moreover, the mechanism provides for significant enhancement of the decay above the contribution that a keV order neutrino mass would provide. Its role in the overall process $\gamma\gamma \to \pi^0 \to \nu\bar{\nu}$ is not yet clear: in [17] we have observed a suppression in $\Gamma(\gamma\gamma \to \pi^0)$ while we have yet to consider the pion propagator modifications in the spirit of [10]. Moreover introduction finite temperature effects is necessary to enable a more detailed application to neutrino emissivity in compact stars.

Acknowledgements

B-Y.P. is grateful for the hospitality of the Special Research Centre for the Subatomic Structure of Matter (CSSM) at the University of Adelaide. J.D.C. thanks the CSSM for support and Tony Williams for informative discussions. A.C.K. is supported by the Australian Research Council.

[1] For a review, see M. Prakash, J. M. Lattimer, R. F. Sawyer, and R. R. Volkas, Annu. Rev. Nucl. Part. Sci. **51** (2001) 295.
[2] E. Fischbach, J. T. Gruenwald, S. P. Rosen, H. Spivack, A. Halprin and B. Kayser, Phys. Rev. **D13**, 1523(1976).
[3] E. Fischbach, S. P. Rosen, H. Spivack, J. T. Gruenwald, A. Halprin and B. Kayser, Phys. Rev. **D16**, 2377(1977).
[4] B. Pontecorvo, J. Expt. Theoret. Phys. (U.S.S.R), **36**, 1615(1959) [translation: Soviet Phys.-JETP **9** 1148(1959)].
[5] H.-Y. Chiu and P. Morrison, Phys. Rev. Lett. **5**, 573(1960).
[6] M. Gell-Mann, Phys. Rev. Lett. **6**, 70 (1961).
[7] D. N. Voskresensky and A. V. Senatorov, JETP Lett. **63**, 885 (1986).
[8] L. B. Leinson, astro-ph/0411025.
[9] W. P. Lam and K.-W. Ng, Phys. Rev. **D44**, 3345(1991).
[10] F. Arretche, A. A. Natale and D. N. Voskresensky, Phys. Rev. **C68**, 0305807 (2003).
[11] P. Jaikumar, M. Prakash and T. Schäfer, Phys. Rev. **D66**, 063003(2002)
[12] S. Reddy, M. Sadzikowski and M. Tachibana, Nucl. Phys. **A714**, 337 (2003).
[13] H.-J. Lee, B.-Y. Park, D.-P. Min, M. Rho and V. Vento, Nucl.Phys. **A723** (2003) 427.
[14] H.-J. Lee, B.-Y. Park, M. Rho and V. Vento, Nucl. Phys. **A726** (2003) 69.
[15] H.-J. Lee, B.-Y. Park, M. Rho and V. Vento, Nucl. Phys. **A741** (2004) 161.
[16] B.-Y. Park, M. Rho and V. Vento, Nucl. Phys. **A 736** (2004) 129.
[17] A.C. Kalloniatis and B.-Y. Park, hep-ph/0410288
[18] T. H. R. Skyrme, Proc. Roy. Soc. Lond. A **260**, 127 (1961).
[19] I. Klebanov, Nucl. Phys. **B262** (1985) 133; L. Castillejo, P.S.J. Jones, A. D. Jackson, J.J.M. Verbaarschot and A. Jackson, Nucl. Phys. **A501** (1989) 801; M. Kugler and S. Shtrikman, Phys. Lett. **B208** (1988) 491; Phys. Rev. **D40** (1989) 3421. A.D. Jackson, A. Wirzba, and L. Castillejo, Nucl. Phys. **A486** (1988) 634; Phys. Lett. **B198** (1987) 315; H. Forkel, A.D. Jackson, M. Rho, and C. Weiss, Nucl. Phys. **A504** (1989) 818; D.I. Diakonov and A.D. Mirlin, Sov. J. Nucl. Phys. **47** (1988) 424.
[20] E. Witten, Nucl. Phys. **B223** (1983) 422.
[21] A. Fetter, J.D. Walecka, "Quantum theory of many-particle systems", Dover (2003).